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PREDICTION OF DYNAMIC CHARACTERISTICS USING UPDATED FINITE ELEMENT MODELS

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Model updating techniques are used to update a finite element model of a structure so that an updated model predicts the dynamics of a structure more accurately. The application of such an updated model in dynamic design demands that it also predict the effects of structural modifications with a reasonable accuracy. This paper deals with updating of a finite element model of a structure and its subsequent use for predicting the effects of structural modifications. Updated models have been obtained by a direct model updating method and by an iterative method of model updating based on the frequency response function (FRF) data. The suitability of updated models for predicting the effect of structural modifications is evaluated by some computer and laboratory experiments. First a study is performed using a simulated fixed–fixed beam. Cases of complete, incomplete and noisy data are considered. Updated models are obtained by the direct and the FRF-based method in each of these cases. These models are then used for predicting the changes in the dynamic characteristics brought about due to a mass and a beam modification. The simulated study is followed by a study involving actual measured data for the case of an F-shape test structure. The updated finite element models for this structure are obtained again by the direct and the FRF-based method. Structural modifications in terms of mass and beam modifications are then introduced to evaluate the updated model for its usefulness in dynamic design. It is found that the predictions based on the iterative method based updated model are reasonably accurate and, therefore, this updated model can be used with reasonable accuracy to perform dynamic design. The predictions on the basis of the direct method based updated model are found to be reasonably accurate in the lower portion of the updating frequency range but the predictions are in a significant error in the remaining portion of the updating frequency range. It is concluded that the updated models that are closer to the structure physically are likely to perform better in predicting the effects of structural modification.

1. INTRODUCTION

Accurate dynamic mathematical model of a structure is essential for simulating reliably the dynamic characteristics. Such a model would allow in improving the dynamic design of a structure at the computer level resulting in an optimized design apart from savings in terms of money and time. In practice, a mathematical model can be derived by analytical approaches such as by finite element method or by experimental approaches such as by modal testing. A mathematical model derived analytically, at times, has been found to be inaccurate especially in the case of complex structures. The discretization error, modelling of joints, boundary conditions and damping and other simplifications made by the analyst in the modelling process could be the possible sources of inaccuracies present in a finite element (FE) model. The experimental approach to extract a model also faces problems due

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to limited number of measured co-ordinates, limited frequency range and difficulty in the measurement of rotational degrees of freedom. While on the one hand a finite-element-based analytical model has the advantage of being complete and precise, on the other the experimental data are generally considered to be more accurate given the availability of reliable data acquisition and measuring equipment and well-developed testing and extraction methods [1, 2]. This has led to the development of model updating which aims at reducing the inaccuracies present in an analytical model in the light of measured dynamic test data while simultaneously allowing to retain a more detailed representation provided by a finite element model. Model updating thus can be seen as an attempt to combine the better aspects of the two approaches.

A number of model updating methods have been proposed in recent years [3–5]. The model updating methods can be broadly classified into direct methods, which are essentially non-iterative ones, and iterative methods. A significant number of methods, [6–8], which were first to emerge, belonged to the direct category. These methods update directly the elements of stiffness and mass matrices and are one-step procedures. The resulting updated matrices though reproduce measured modal data exactly but do not generally maintain structural connectivity and the corrections suggested are not physically meaningful.

Iterative methods have generally been based on either modal data or frequency response function (FRF) data. The modal data based iterative method proposed in reference [9] is quite popular due to the freedom it allows in the choice of the updating parameters and the applicability of the method even with an incomplete data. A model updating approach has been proposed recently, [10], which is based on framing the updating problem as a constrained non-linear optimization problem. There have been attempts to use directly the measured FRF data for updating as in reference [11]. In references [12, 13] several studies have been conducted using simulated and experimental data to gauge the effectiveness of this FRF-based iterative technique. Recently, genetic algorithms have also been employed for model updating [14, 15]. The selection of variables to be updated is very important in finite element model updating, as the ultimate goal is to minimize the modelling error present in the model. Generic element matrices giving rise to the choice of parameters that allowed for changes in the structure of the mass and stiffness matrices by modifying the eigenvalues and eigenvectors of individual finite elements are introduced in reference [16]. The generic element formulation is applied to the problem of joint identification in reference [17] while in reference [18] a strategy is proposed for the parameterization of a welded joint and a clamped end. In reference [19], an experiment-based finite element model has been used for evaluating the effects of design changes.

A model updating method has been generally evaluated on the basis of how closely the dynamic characteristics of the resulting updated model approximate the measured dynamic test data. But it needs to be investigated as to whether an updated model is capable of predicting the changes in the dynamic characteristics of a structure due to potential structural modifications with reasonable accuracy. This capability in an updated model is essential for carrying out a reliable dynamic design, which happens to be one of the important applications where an updated model can be used. Very little appears to have been done on this aspect and this forms the subject of the present paper. This paper deals with updating of a finite element model of a structure and its subsequent use for predicting the effects of structural modifications. Updated models have been obtained by a direct model updating method and by an iterative method of model updating based on the frequency response function data. The suitability of updated models for predicting the effect of structural modifications is evaluated by some computer and laboratory experiments. First, a study is performed using a simulated fixed–fixed beam. Cases of complete, incomplete and noisy data are considered. Updated models are obtained by both the methods in each of these cases. These models are then used for predicting the changes in the dynamic characteristics brought about due to a mass and then a beam modification. This is followed by a study involving actual measured data for the case of an F-shape test structure. Structural modifications in terms of mass and beam modifications are then introduced to evaluate the updated models for its usefulness in dynamic design.

2. THEORY

Two methods of model updating one of which is a direct method, the method proposed in reference [7], and the other is an iterative method based on FRF data, proposed in reference [11] and referred to as response function method, have been employed in this work for obtaining updated models. The basic theory of these methods is briefly presented here. In the case of response function method, the physical parameters of the model are proposed to be used as updating variables.

2.1. DIRECT METHOD

In this method, [7], the updating of the FE model is performed in two steps. In the first step, the analytical mass matrix is updated subject to the orthogonality constraint. The corrections to the FE model mass matrix $[M_A]$ are made such that the updated mass matrix $[M_U]$ is as close as possible, in some sense, to the analytical mass matrix. The problem is stated as that of finding $[M_U]$ which minimizes objective function J given by

$$J = \tfrac{1}{2}\|[M_A]^{-1/2}([M_U] - [M_A])[M_A]^{-1/2}\| \tag{1}$$

such that the measured eigenvector matrix $[\phi_m]$ and $[M_U]$ satisfy the orthogonality constraint

$$[\phi_m]^{\mathrm{T}}[M_U][\phi_m] = [I], \tag{2}$$

where $[I]$ is a unity matrix. The above constrained minimization problem is converted to an equivalent unconstrained minimization problem by constructing the Lagrangian function, which incorporates equality constraints into it using Lagrange mutipliers. The unknown updated mass matrix that minimizes this Lagrangian function is obtained as

$$[M_U] = [M_A] + [M_A][\phi_m]\overline{[M_A]}^{-1}([I] - \overline{[M_A]})\overline{[M_A]}^{-1}[\phi_m]^{\mathrm{T}}[M_A], \tag{3}$$

where

$$\overline{[M_A]} = [\phi_m]^{\mathrm{T}}[M_A][\phi_m]. \tag{4}$$

In the second step, the analytical stiffness matrix $[K_A]$ is updated. Again, the corrections to the stiffness matrix are made such that the updated stiffness matrix $[K_U]$ is as close as possible, in some sense, to the analytical stiffness matrix. The problem is stated as that of finding $[K_U]$ that minimizes

$$J = \tfrac{1}{2}\|[M_U]^{-1/2}([K_U] - [K_A])[M_U]^{-1/2}\| \tag{5}$$

subject to the constraints that the updated stiffness matrix satisfies the equation of motion of the structure and that it be symmetric. The constraints can be stated as

$$[K_U][\phi_m] = [M_U][\phi_m][\Lambda_m] \tag{6}$$

and

$$[K_U] = [K_U]^{\mathrm{T}}, \tag{7}$$

where $[\Lambda_m]$ is a diagonal matrix of the measured eigenvalues. The solution of the above problem yields an expression for the updated stiffness matrix given by

$$\begin{aligned}[K_U] = {} & [K_A] - [K_A][\phi_m][\phi_m]^{\mathrm{T}}[M_U] - [M_U][\phi_m][\phi_m]^{\mathrm{T}}[K_A] \\ & + [M_U][\phi_m][\phi_m]^{\mathrm{T}}[K_A][\phi_m][\phi_m]^{\mathrm{T}}[M_U] + [M_U][\phi_m][\Lambda_m][\phi_m]^{\mathrm{T}}[M_U].\end{aligned} \tag{8}$$

It can be noted that since practically the mode shape data is incomplete, to implement the above equations, either FE model matrices are to be reduced or mode shapes are to be expanded. In the study reported here, the mode shapes are expanded so that the updated FE model is a full size model. This offers ease in introducing a structural modification at the analytical level. The method of system equivalent reduction expansion process (SEREP), [20], which makes use of the FE model modal data to expand the mode shapes has been used.

2.2. RESPONSE FUNCTION METHOD (RFM)

This method, [11], which is an iterative method uses measured FRF data directly without requiring any modal extraction to be performed. The identities relating dynamic stiffness matrix $[Z]$ and receptance FRF matrix $[\alpha]$ can be written for the analytical model and corresponding to the actual structure, respectively, as follows:

$$[Z_A][\alpha_A] = [I], \tag{9}$$

$$[Z_X][\alpha_X] = [I], \tag{10}$$

where subscripts A and X denote analytical or FE model and experimental model respectively. Expressing $[Z_X]$ in equation (10) as $[Z_A] + [\Delta Z]$ and then subtracting equation (9) from it, the following matrix equation is obtained:

$$[\Delta Z][\alpha_X] = [Z_A]([\alpha_X] - [\alpha_A]). \tag{11}$$

Premultiplying the above equation by $[\alpha_A]$ and then using equation (9) gives

$$[\alpha_A][\Delta Z][\alpha_X] = [\alpha_X] - [\alpha_A]. \tag{12}$$

If only the jth column of measured FRF matrix $[\alpha_X]$, $\{\alpha_X\}_j$, is available then the above equation is reduced to

$$[\alpha_A][\Delta Z]\{\alpha_X\}_j = \{\alpha_X\}_j - \{\alpha_A\}_j, \tag{13}$$

which is the basic relationship of the response function method. A physical variables based updating parameter formulation is used in the present study. Let $\{p\} = \{p_1, p_2, \ldots, p_{nu}\}$ be the vector of physical variables associated with individual or group of finite elements.

Linearizing $[\Delta Z]$ with respect to $\{p\}$ gives

$$[\Delta Z] = \sum_{i=1}^{nu} \left(\frac{\partial [Z]}{\partial p_i} \Delta p_i \right). \tag{14}$$

For an undamped model $[\Delta Z]$ is replaced by $[\Delta K] - \omega^2[\Delta M]$. On dividing and multiplying the above equation by p_i and then writing u_i in place of $\Delta p_i/p_i$, the equation becomes

$$[\Delta Z] = \sum_{i=1}^{nu} \left(\frac{\partial([\Delta K] - \omega^2[\Delta M])}{\partial p_i} p_i \right) u_i \cdot \tag{15}$$

Thus, $\{u\} = \{u_1, u_2, \ldots, u_{nu}\}$ is the unknown vector of fractional correction factors to be determined during updating. Equation (13), after making substitution for $[\Delta Z]$ from equation (15), can be written at various frequency points chosen from the frequency range of interest. The resulting equations can be framed in the following matrix form:

$$[C(\omega)]_{(n \times nf) \times (nu)} \{u\}_{nu \times 1} = \{\Delta\alpha(\omega)\}_{(n \times nf) \times 1} \cdot \tag{16}$$

Practically, the FRFs are available only at a few degrees of freedom thereby rendering the FRF data incomplete. In the present study, the co-ordinate incompleteness has been dealt by using analytically generated FRFs. This has been done by replacing the responses of unmeasured co-ordinates by their analytical counterparts. Once equation (16) has been framed, the equations corresponding to such unmeasured co-ordinates are then removed from it. The equations are then solved for $\{u\}$ in a least square way. The $\{u\}$ so found is used to update vector of physical variables $\{p\}$ and then the updated version of the analytical finite element model is built using these new sets of physical variables. This process is repeated in an iterative way until convergence is obtained. It should be noted that the $\{u\}_j$ found in jth iteration represents the vector of fractional correction factors for the current $\{p\}$, i.e., $\{p\}_j$, and does not represent a cumulative correction with respect to the $\{p\}$ existing before updating, the $\{p\}_0$. On this basis at the end of jth iteration the ith cumulative fractional correction factor is given by

$$\bar{u}_i^j = (1 + u_i^1)(1 + u_i^2) \ldots (1 + u_i^j) - 1 . \tag{17}$$

3. STRUCTURAL MODIFICATION USING AN UPDATED MODEL

An updated undamped finite element model for a structure is available in terms of a stiffness matrix and a mass matrix denoted by $[K]$ and $[M]$ respectively. If $[\Delta K]$ and $[\Delta M]$ represent the modification matrices due to a modification then the modified structure's stiffness and mass matrix denoted by $[K_m]$ and $[M_m]$, respectively, can be written as

$$[K_m] = [K] + [\Delta K], \tag{18}$$

$$[M_m] = [M] + [\Delta M]. \tag{19}$$

Consider the case of mass modification by assuming that a mass m_0, kg is added at ith node. The $[\Delta M]$ is obtained by making the diagonal entries corresponding to the translational degrees of freedom for the ith node equal to " $+ m_0$" assuming that the rotary inertia of the modification is negligible [21]. The mass modification matrix is given as

$$[\Delta M] = \begin{bmatrix} 0 & \cdots & \cdots & \cdots & \cdots & 0 \\ \vdots & \ddots & & & & \vdots \\ \vdots & & +m_0 & & & \vdots \\ \vdots & & & +m_0 & & \vdots \\ \vdots & & & & \ddots & \vdots \\ 0 & \cdots & \cdots & \cdots & \cdots & 0 \end{bmatrix}. \tag{20}$$

For the case of mass modification the stiffness matrix remains unaffected.

For the case of beam modification, the $[K_m]$ and $[M_m]$ are essentially obtained by assembling the FE model for the added beam member with that of the FE model of the unmodified structure. Predictions on the basis of the updated model can be made by assembling the FE model for the added beam member with that of the updated FE model of the unmodified structure. Thus, in general, the number of finite elements, the number of nodes and consequently the size of the modified model will be higher than that for the unmodified model. In terms of equations (18) and (19), the $[K]$ and $[M]$ represent the structural matrices, expanded to the size of the modified model, corresponding to either updated or unupdated FE model depending upon which model is made the basis for making predictions. The modification matrices $[\Delta K]$ and $[\Delta M]$ represent the FE model for the added beam-member expanded to the size of the modified model. It can be noted that for the case of beam modification, both the mass and stiffness matrices are affected.

Once for a given modification the $[K_m]$ and $[M_m]$ are established via equations (18) and (19) the eigenvalues, $[\lambda_m]$, and eigenvectors, $[\phi_m]$, of the modified structure predicted by a model can be obtained by resolving the following eigenvalue problem:

$$[K_m][\phi_m] = [M_m][\phi_m][\lambda_m]. \tag{21}$$

4. PREDICTION OF DYNAMIC CHARACTERISTICS OF A BEAM STRUCTURE USING SIMULATED DATA BASED UPDATED MODELS

A simulated study on a fixed–fixed beam is conducted for evaluating the suitability of updated models for dynamic design. The dimensions of the beam are $9100 \times 50 \times 5$ mm.The modulus of elasticity and density are taken as 2·0E + 11 N/m^2 and 7800 kg/m^3 respectively. The beam is modelled using 30 beam elements with node at ends fixed giving a total of 29 nodes with three degrees of freedom (two displacements and one rotation) each. The simulated modal and FRF data, which are treated as experimental data, are obtained by generating a finite element model by introducing certain known discrepancies in the thickness of all the finite elements with respect to the analytical model, the details of which are given in Table 1. The frequency range from 0 to 1 kHz covering seven modes is taken as the measured frequency range.

TABLE 1

Discrepancies between the finite element and the simulated experimental model

Element number	3	5	11	16	25	29	All other elements
% deviation in thickness	+20%	+40%	+25%	+40%	+30%	+30%	+10%

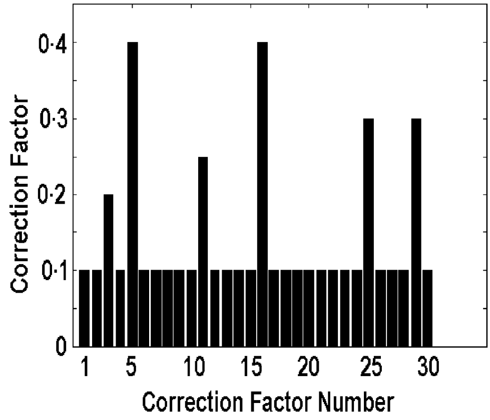

Figure 1. The fractional correction factors to the updating parameters obtained using the response function method for the case of *complete data.*

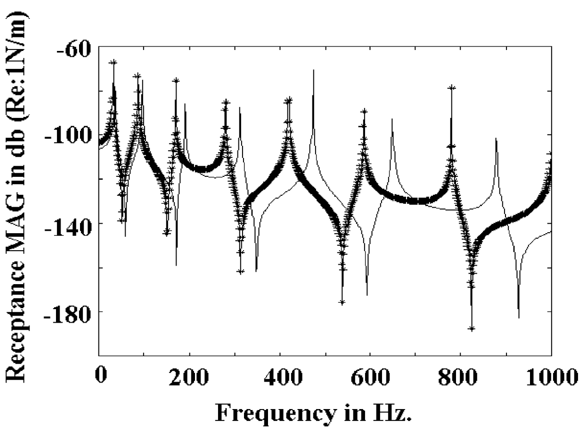

Figure 2. Overlay of the simulated experimental FRF(——) and the FE model FRF(***) before updating.

In the case of updating based on the response function method using simulated data, the individual thickness of all the finite elements are taken as the updating parameters. The frequency points used in framing the updating equations span a frequency range that covers the first five modes of the structure. For the case of updating based on the direct method also, eigendata corresponding to the first five modes of the structure have been utilized.

First the case of a complete data is considered where it is assumed that all the degrees of freedom of the finite element model are measured ones. Thus, in this case one complete column of the FRF matrix and all the eigenvalues and the corresponding eigenvectors falling in the measurement frequency range will be known. Figure 1 gives the fractional correction factors to the updating parameters obtained using the response function method. The identified correction factors are found to be exactly identical to the introduced discrepancies. An overlay of the simulated experimental FRF ($\alpha 11y + 5y +$) and the FE model FRF ($\alpha 11y + 5y +$) before updating is shown in Figure 2. Figure 3 compares the overlays of the simulated experimental FRF and the FE model FRF after updating for the case of complete data obtained by the response function method and the direct method. It is

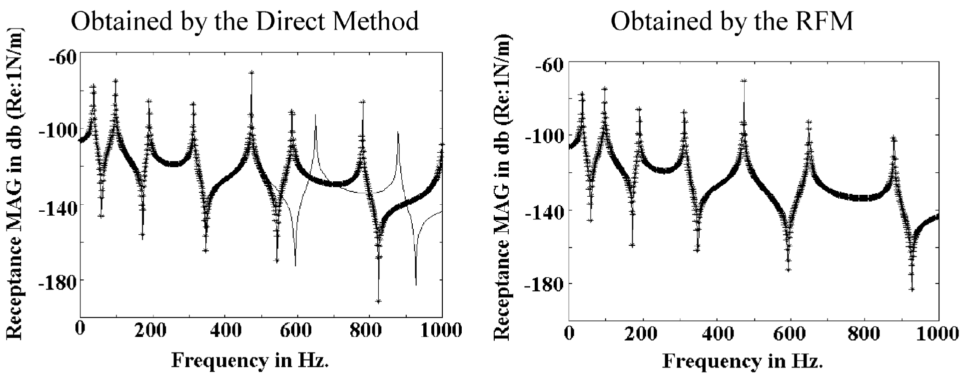

Figure 3. Comparison of the overlays of the simulated experimental FRF (——) and the FE model FRF (***) after updating for the case of *complete data* obtained by the response function method and the direct method.

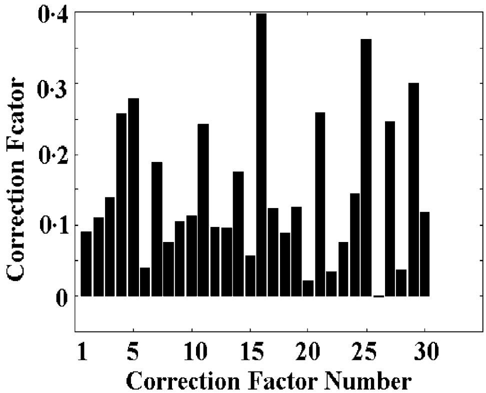

Figure 4. The fractional correction factors to the updating parameters obtained using the response function method for the case of *incomplete and noisy data*.

observed that within the updating range covered by the first five modes, both the methods have produced a very good FRF-fit. But outside the updating range, there is a considerable lack of fit in the case of direct method.

In practice, it is not realistic that all the co-ordinates specified in the analytical FE model have been measured either due to physical inaccessibility or due to difficulties faced in the measurement as that for rotational degrees of freedom. The second case considered, therefore, is that of an incomplete measured data. It is assumed that only lateral degree of freedom has been measured at 15 alternate nodes leaving 82.7% degrees of freedom unmeasured. The incompleteness of the FRF-data is dealt with as explained in the last section. The eigenvectors are expanded by the SEREP method using the first seven FE model modes. It is observed that the results obtained are almost the same as that for the complete case which means that both the methods have performed well in the presence of an incomplete data also.

In practical measurement, the measured FRFs will be contaminated by measurement noise and consequently the extracted modal parameters will also be affected. In the third case the FRFs, the natural frequencies and the mode shapes corresponding to simulated experimental data were polluted with random errors. While the incomplete FRFs and the mode shapes were polluted with 2% random noise, the natural frequencies were polluted with 0.2% noise. Figure 4 gives the fractional correction factors to the updating parameters

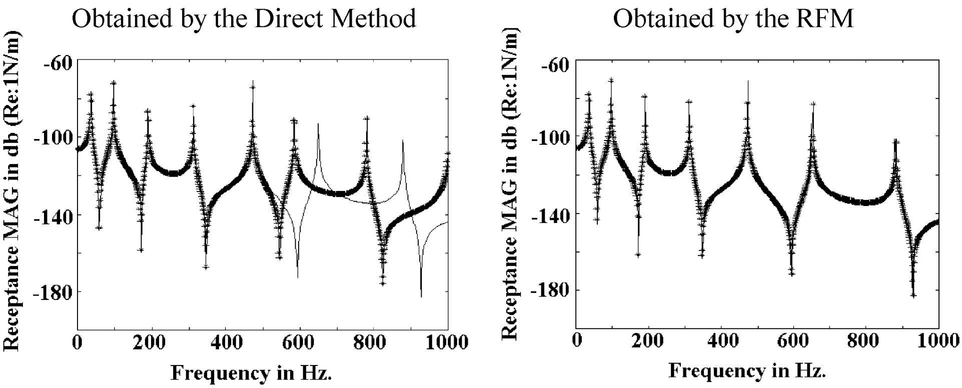

Figure 5. Comparison of the overlays of the simulated experimental FRF (——) and the FE model FRF (***) after updating for the case of *incomplete and noisy data* obtained by the response function method and the direct method.

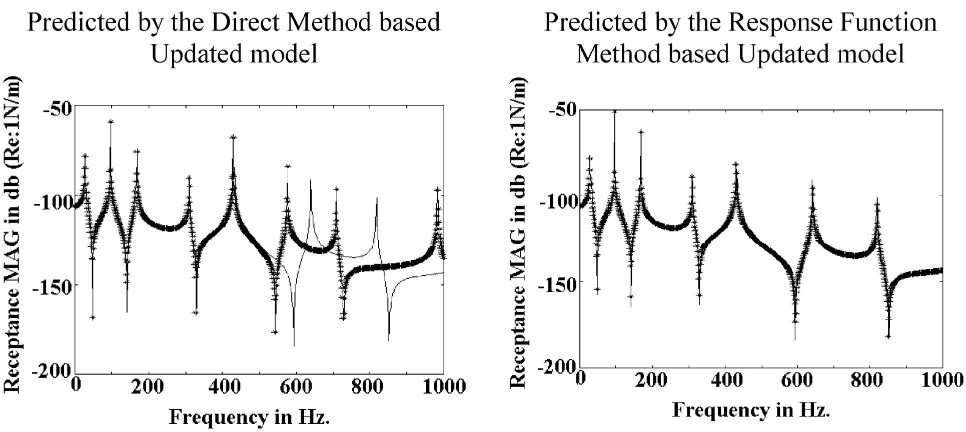

Figure 6. Comparison of the overlay of the FRF of the *mass-modified structure* as predicted by the response function method based and the direct method based updated model. (Exact modified FRF (——) overlaid on the modified FRF (***) predicted by an updated model).

obtained using the response function method while Figure 5 gives a comparison of the FRF-overlay obtained by the two methods. After comparing Figure 4 with Figure 1 it is seen that the correction factors have been identified reasonably though not very accurately. Again, the FRF-fit beyond the updating range is not good for the case of direct method based updated model.

Two cases of structural modification are now considered to evaluate the suitability of updated models for dynamic design. The first case is that of a mass modification in terms of an addition of a lumped mass of 0·5 kg at the node number 16 which is around the middle of the beam. The modified mass is accounted for in the analytical model as described in the previous section. Results for the case of incomplete and noisy data only are given here as the results for this case are found to be very close to the results for the cases of complete and incomplete data. Figure 6 compares the FRF of the mass-modified structure as predicted by the response function method based and the direct method based updated model. For evaluation purpose, the updated models based modified FRFs have been overlaid on the

TABLE 2

Comparison of the natural frequency and the mode shape correlation for the mass-modified structure as predicted by the updated models based on the two methods

Mode no.	Simulated model based frequency for the modified structure (Hz)	Predictions for the modified structure on the basis of the direct method based updated model			Predictions for the modified structure on the basis of the response function method based updated model		
		Frequency (Hz)	% Error	MAC-value	Frequency (Hz)	% Error	MAC-value
1	27·88	27·92	0·13	1·0	27·91	– 0·09	0·9999
2	96·30	96·30	0·0	0·9999	96·27	– 0·03	0·9999
3	167·83	167·73	0·05	0·9999	167·44	– 0·23	0·9998
4	310·56	310·50	0·02	0·9999	310·41	– 0·04	0·9998
5	431·81	428·84	– 0·68	0·9940	430·45	– 0·31	0·9995
6	639·82	576·10	– 9·95	0·9978	641·73	0·29	0·9988
7	819·67	708·54	– 13·55	0·9931	820·52	0·10	0·9983

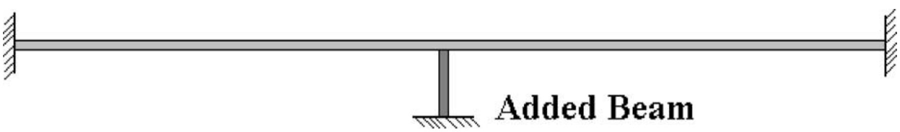

Figure 7. Beam structure with beam modification.

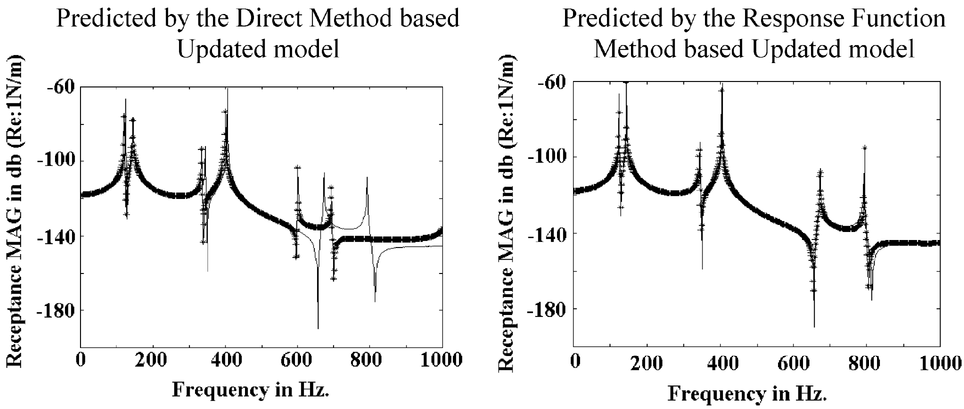

Figure 8. Comparison of the overlay of the FRF of the *beam-modified structure* as predicted by the response function method based and the direct method based updated model. (Exact modified FRF(——) overlaid on the modified FRF(***) predicted by an updated model).

actual modified FRF. The actual modified FRF can always be obtained analytically in a simulated study, as the discrepancies introduced to generate simulated data are exactly known. It is observed from the comparison that the modified FRF predicted by the RFM -based updated model is almost identical to the actual modified FRF. The modified

TABLE 3

Comparison of the natural frequency and the mode shape correlation for the beam-modified structure as predicted by the updated models based on the two methods

Mode no.	Simulated model based Frequency for the modified structure (Hz)	Predictions for the modified structure on the basis of the direct method based updated model			Predictions for the modified structure on the basis of the response function method based updated model		
		Frequency (Hz)	% Error	MAC-value	Frequency (Hz)	% Error	MAC-value
1	123·85	119·83	− 3·24	0·9973	123·54	− 0·25	0·9995
2	144·15	144·37	− 0·15	0·9978	143·67	− 0·33	0·9999
3	344·20	334·68	− 2·76	0·9973	343·29	− 0·26	0·9998
4	406·10	399·74	− 1·56	0·9864	403·83	− 0·55	0·9997
5	673·20	601·12	− 10·70	0·9647	673·17	0·0	0·9985
6	791·13	693·35	− 12·35	0·9589	793·69	0·32	0·9978
7	1138·68	1008·79	− 11·40	0·9748	1134·77	− 0·34	0·9917

FRF predicted by the direct method based updated model is very close to the actual FRF within the updating range but beyond it, there is a significant error in the fit. A comparison of the natural frequency and the mode shape correlation as predicted by the updated models based on the two methods is also shown in Table 2.

The second case, a little more complex, is that of a beam modification. A beam member of length 0·06 m, of the same cross-section as that of the unmodified beam, is added at node number 16 as shown in Figure 7. The added beam member is modelled by two finite elements and is accommodated in the analytical model as described in the previous section. Again, the results for the case of incomplete and noisy data only are given here as the results for this case are found to be very close to the results for the cases of complete and incomplete data. Figure 8 shows a comparison of the modified FRFs as predicted by the updated models based on the two methods. A comparison of the natural frequency and the mode shape correlation as predicted by the updated models based on the two methods is also shown in Table 3. It is observed that the prediction of the modified characteristics has been very good on the basis of the updated model obtained by RFM. The predictions based on the direct method based updated model have been in the significant error beyond the updating frequency range. Within the frequency range also the error in the fifth mode is on the higher side. In the present study, the results of prediction of modified characteristics based on the RFM based updated model are found to be accurate since the identified correction factors after updating were very well matching with the introduced discrepancies. This means that only physically meaningful corrections were made to the FE model at the updating stage. To put it in a more precise way the corrections were made, in terms of their locations and amount, only where they were actually required. The corresponding situation for the case of direct method with regard to the locations and the amount of corrections that were made in comparison to where they were actually required is depicted in Figures 9 and 10. Figure 9 compares plots of the difference matrices $(K_X - K_A)$ and $(K_U - K_A)$, while Figure 10 compares plots of the difference matrices $(M_X - M_A)$ and $(M_U - M_A)$. $(K_X - K_A)$ represents the simulated errors in the FE stiffness matrix. Thus, the corresponding picture represents the locations and the magnitudes of inaccuracies in the FE stiffness matrix. $(K_U - K_A)$ represents the location and the amount of corrections that were made to the FE

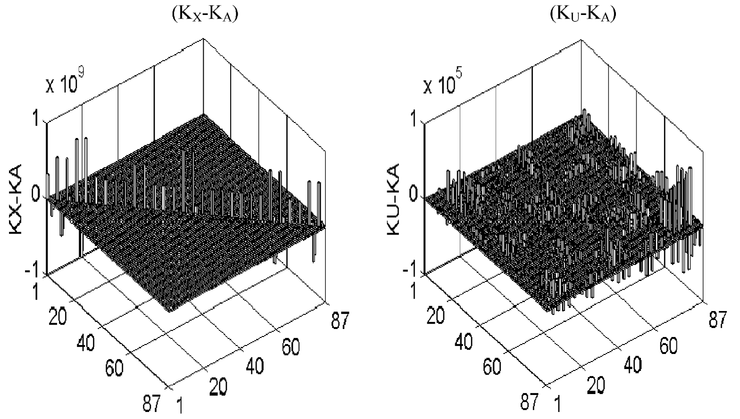

Figure 9. A comparison of the plots of the difference matrices $(K_X - K_A)$ and $(K_U - K_A)$.

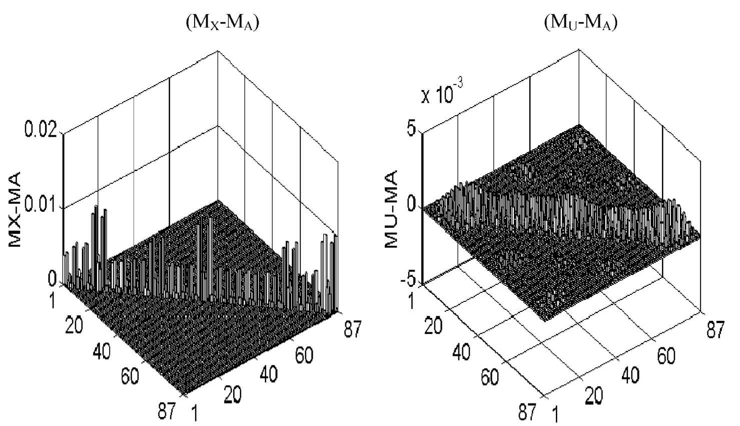

Figure 10. A comparison of the plots of the difference matrices $(M_X - M_A)$ and $(M_U - M_A)$.

stiffness matrix. Similarly, $(M_X - M_A)$ and $(M_U - M_A)$ represent the corresponding quantities for the FE mass matrix. These figures indicate that the changes made in the FE model matrices in the case of updating using direct method, to achieve a match between the test and the updated FE modal data, have not been able to minimize significantly the modelling inaccuracies. This seems to be the reason why the direct method based updated model predictions are poorer than the RFM based updated model predictions. On the basis of these results, therefore, it can be concluded that the updated models that are closer to the structure physically are likely to perform better in predicting the effects of structural modification.

5. PREDICTION OF DYNAMIC CHARACTERISTICS OF AN F-SHAPE STRUCTURE USING EXPERIMENTAL DATA BASED UPDATED MODELS

The suitability of updated models for dynamic design is now evaluated for the case of an F-shape structure, shown in Figure 11, using experimental data. The F-shape structure has

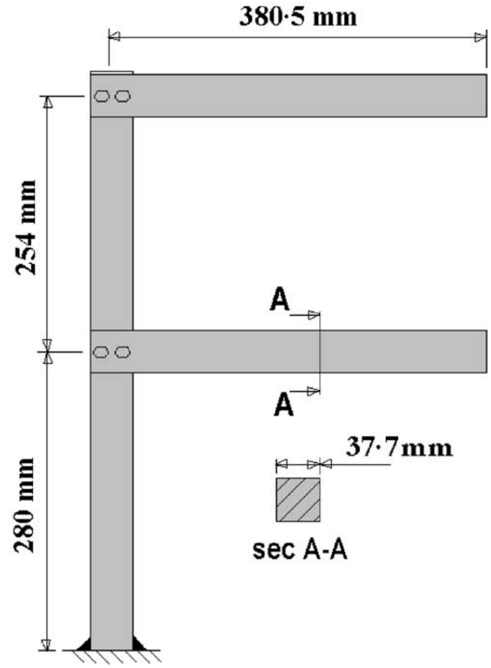

Figure 11. F-shape structure.

been constructed by bolting horizontally the two beam members to a vertical beam member, which in turn has been welded to a base plate at the bottom. All the beam members have a square cross-section with 37·7 mm as one of its side. A finite element model of the structure is built using 48 2-D-frame elements with three degrees of freedom (two displacements and one rotation) at each of the nodes. The values for the modulus of elasticity and the density are taken as $2{\cdot}0e + 11\ \mathrm{N/m^2}$ and $7800\ \mathrm{kg/m^3}$ respectively. An undamped eigenvalue problem is set and solved in order to obtain an analytical estimate of undamped natural frequencies and mode shapes. The modal test is performed by exciting the structure with an impact hammer at 16 locations and measuring the response by an accelerometer kept fixed at one of the locations. The frequency response functions so acquired are analyzed using a global curve fitting method available in ICATS [22] to obtain an experimental set of modes in the range of 0–1600 Hz.

The correlation between the analytical and the experimental set of modal data is now performed using modal assurance criterion (MAC) [23]. On the basis of MAC-matrix, the correlated mode pairs are established and then the existing level of differences between the corresponding natural frequencies are ascertained. The results of such an exercise carried out for the above case are shown in Table 4 indicating corresponding experimental and analytical natural frequencies, percentage difference between them and the corresponding MAC-value for the seven modes. An overlay of the measured FRF24 × 17 × and the corresponding FE model FRF is also shown in Figure 12. It is observed that though mode shape correlation is reasonably good, the error in the analytical natural frequencies is significantly high.

To improve the correlation of the analytical modal data with the experimental data the FE model is now updated. First the updating is carried out by the direct method using the first five modes in the updating process. Before updating, the modes are expanded by the

TABLE 4

Correlation of measured and FE-model based modal data

Mode no.	Measured frequency in (Hz)	FE model predictions		MAC-value
		Frequency in (Hz)	% Error	
1	34·95	43·05	23·17%	0·9901
2	104·02	123·67	18·89%	0·9470
3	133·96	185·21	38·26%	0·9265
4	317·52	385·17	21·30%	0·9054
5	980·16	1020·06	4·07%	0·7299
6	1057·8	1084·79	2·55%	0·8040
7	1531·45	1925·76	25·74%	0·8798

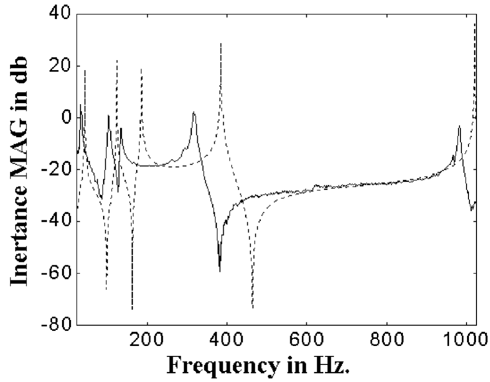

Figure 12. Overlay of the measured FRF 24 × 17 × (——) and the corresponding FE model FRF (-----) before updating.

SEREP method using nine analytical modes. The updating is then carried out by the response function method. The choice of updating parameters on the basis of engineering judgement with respect to the possible locations of modelling error in structure is one of the strategies that can possibly ensure that only meaningful corrections are made. Such a selection of updating parameters is possible since an iterative method, like RFM, allows such a choice to be made. In the present case due to the presence of three joints, the modelling of stiffness at these places is expected to be a dominant source of inaccuracy in the FE model assuming that the values of the material and the geometric parameters are correctly known. The three joints are modelled by taking coincident nodes at each of them. Torsional springs of stiffness $Kt1$, $Kt2$ and $Kt3$ coupling the rotational degrees of freedom of the coincident nodes at the three joints are taken as updating parameters. The other two degrees of freedom of the coincident nodes are taken as rigidly coupled. Since an undamped FE model is being updated, the FRF corresponding to the FE model has only a real part while the measured FRF has both the real as well as the imaginary part. Figure 13 shows an overlay of the measured FRF 24 × 17 × and the real part of the same FRF. It is seen that the two response functions are almost identical except in the regions very close to the resonances and antiresonances. Therefore, for the frequency range excluding these regions the contribution of the imaginary part to the measured FRF is very small. In the light of this

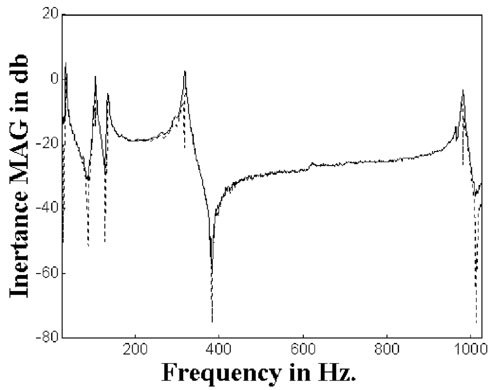

Figure 13. Overlay of the measured FRF 24 × 17 × (——) and its real part (----).

TABLE 5

Corrected values of the updating variables

Updating variable	Initial value	Final value
*Kt*1	3·28E + 06	2·61E + 05
*Kt*2	3·28E + 06	2·95E + 05
*Kt*3	3·28E + 06	3·00E + 05

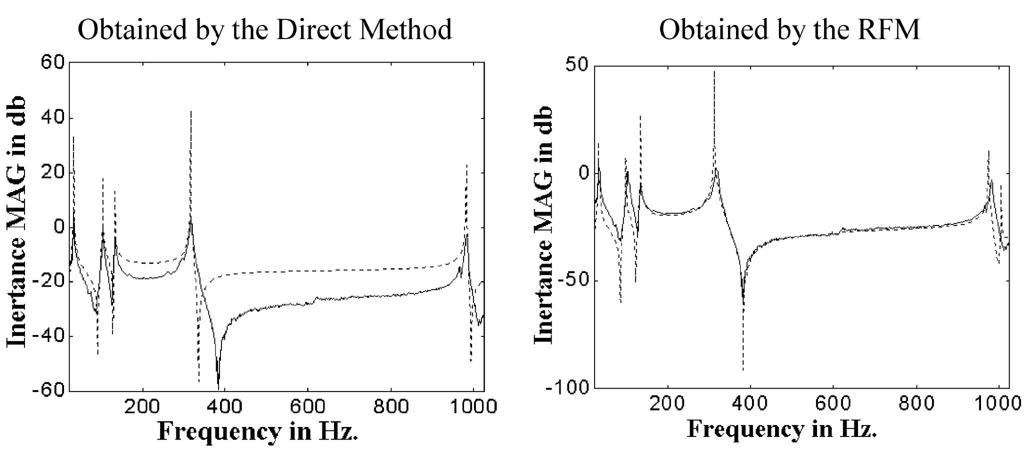

Figure 14. Comparison of the overlays of the experimental FRF (——) and the updated model FRF (----) after updating obtained by the response function method and the direct method.

observation, the imaginary part of the measured FRFs is set to zero for the purpose of making updating calculations. The frequency points to be used in updating are selected spanning a frequency range covering the first five modes by ensuring that the points do not lie very close to the resonances and the antiresonances.

Table 5 gives the initial values and the final values of the updating parameters as obtained in the case of RFM. A comparison of the direct method based updated model FRF and the RFM based updated model FRF is shown in Figure 14. It is seen that the RFM based

Table 6

Comparison of the correlation between the measured and the updated models natural frequencies and the mode shapes

Mode no.	Measured frequency (Hz)	Dynamic characteristics of the direct method based updated model			Dynamic characteristics of the response function method based updated model		
		Frequency (Hz)	% Error	MAC-value	Frequency (Hz)	% Error	MAC-value
1	34·95	34·95	0·0	1·0	34·25	– 2·00	0·9923
2	104·02	104·02	0·0	1·0	100·27	– 3·60	0·9693
3	133·96	133·96	0·0	1·0	134·42	0·34	0·9675
4	317·52	317·52	0·0	1·0	313·73	– 1·19	0·9423
5	980·16	980·16	0·0	1·0	973·44	– 0·68	0·4370
6	1057·8	1042·64	– 1·43	0·8649	1004·53	– 5·03	0·5639
7	1531·45	1921·18	25·44	0·8540	1495·59	– 2·34	0·9669

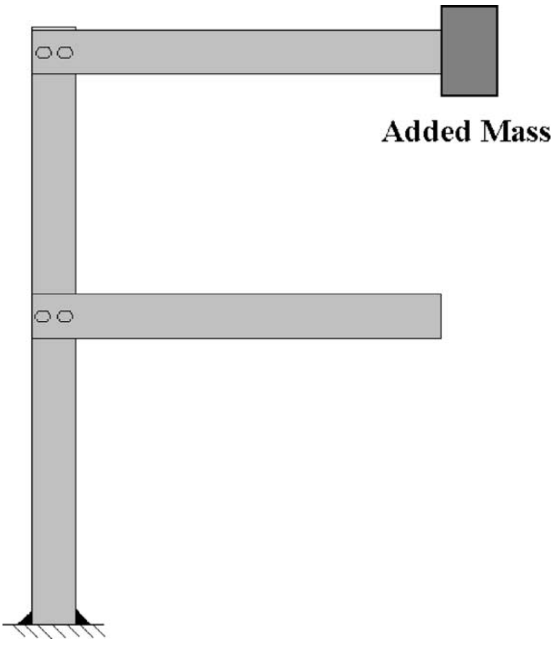

Figure 15. F-shape structure with mass modification.

updated model FRF has a very good match with the measured FRF. The direct method based updated model FRF also has a good fit in the lower frequency range but has a considerable deviation in the higher frequency range away from the resonances. A comparison of the correlation between the measured and the updated model natural frequencies and the mode shapes is also given in Table 6. As is characteristic of the direct method, the modal data used for updating has been exactly reproduced by the updated model. For the case of the RFM -based updated model, there is also a significant reduction in the average natural frequency error for the modes beyond the updating range.

The updated models obtained above are now used for predicting the effects of potential design modifications made to the structure. The predictions are made first for a mass modification and then for a beam modification.

A mass modification is introduced by attaching a mass of 1·8 kg at the tip of the upper horizontal beam member as shown in Figure 15. The FRFs for the mass-modified structure are then acquired. The mass modification is also introduced analytically in the updated models obtained by the two methods as explained above. The mass and stiffness matrix for the modified structure, and subsequently its modal data and the FRFs, corresponding to the updated models are obtained as explained in section 3. A comparison of the modified FRFs as predicted by the updated models based on the two methods is shown in Figure 16 while a comparison of the predicted modal data for the modified structure is given in Table 7. It is observed that the predicted dynamic characteristic on the basis of the RFM based updated model are much closer to the measured characteristics for the modified structure then the direct method based updated model.

A beam modification is introduced in the form of a stiffener of width 38·2 mm and thickness 5 mm.The stiffener is attached between the tips of the lower and the upper horizontal beam members as shown in Figure 17. The modal data for the modified structure is obtained by performing a modal test on the modified structure. The prediction of FRFs on the basis of the two updated models are compared in Figure 18 while Table 8 gives a comparison of the predicted natural frequencies and the mode shape correlation.

The frequency response curve predicted by the RFM based updated model is following the measured curve quite closely while the curve predicted by the direct method based updated model is deviating significantly. It is also noted that the beam modification has drastically altered the dynamic characteristics of the structure as compared to the case of mass modification.

In terms of the errors in the predicted natural frequencies for the first five modes, falling in the updating frequency range, the results discussed above can be summarized as follows. The % average errors in the natural frequencies in the updated models based on the direct method and the response function method are 0·0 and 1·56% respectively. For the case of mass modification, the % errors in natural frequency prediction for the modified structure are 6·56 and 2·84% by the direct method and the RFM respectively. For the case of beam

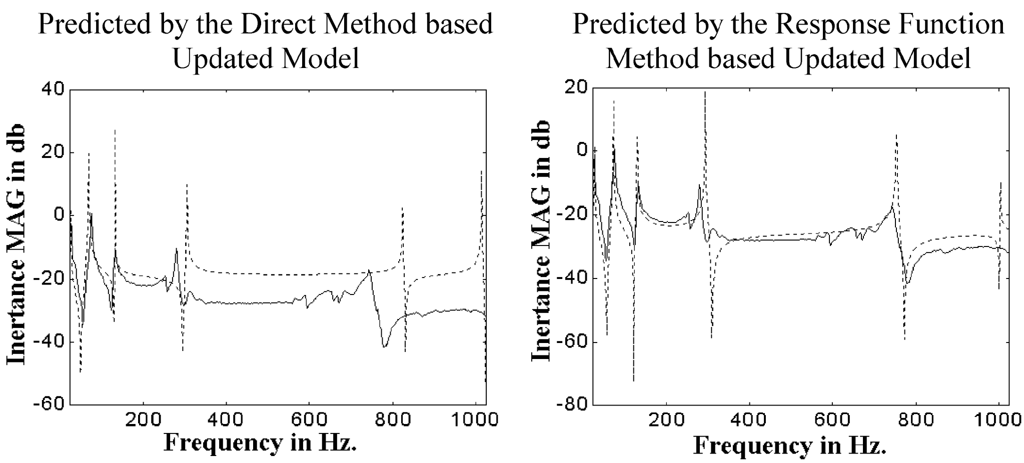

Figure 16. A comparison of the overlays of the measured modified FRF (——) and those predicted by the updated models (-----) obtained by the direct method and the response function method for the case of *mass modification*.

TABLE 7

Comparison of the natural frequency and the mode shape correlation for the mass-modified structure as predicted by the updated models based on the two methods

Mode no.	Measured frequency for the modified structure (Hz)	Predictions for the modified structure on the basis of the direct method based updated model			Predictions for the modified structure on the basis of the response function method based updated model		
		Frequency (Hz)	% Error	MAC-value	Frequency (Hz)	% Error	MAC-value
1	27·32	25·80	− 5·54	0·9791	28·45	4·13	0·9856
2	74·53	69·01	− 7·40	0·9579	72·38	− 2·87	0·9950
3	133·38	133·74	0·27	0·9965	131·58	− 1·34	0·9926
4	280·11	304·67	8·76	0·6074	293·65	4·83	0·7622
5	745·12	825·73	10·81	0·7277	753·01	1·05	0·6482
6	1050·47	1014·16	− 3·45	0·9842	1003·21	− 4·49	0·9843
7	1522·66	1878·26	23·35	0·8371	1478·67	− 2·88	0·9658

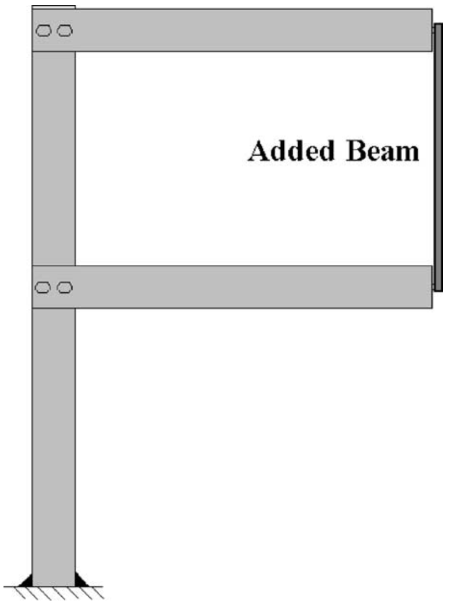

Figure 17. F-shape structure with beam modification.

modification this errors are 8·18 and 2·73% for the two methods respectively. Thus it is observed that the % average prediction errors for the modified structures on the basis of RFM-based updated model is of around the same order as that existing in the corresponding updated model. But in the case of the direct method though the % natural frequency error in the updated model is 0.0%, the % average prediction errors for the modified structures are much higher. With regard to the FRF-prediction also, the predictions on the basis of the RFM based updated models seem to be following the corresponding measured FRF much closer than the prediction on the basis of the direct method based updated model. These observations are on the similar lines as were seen in

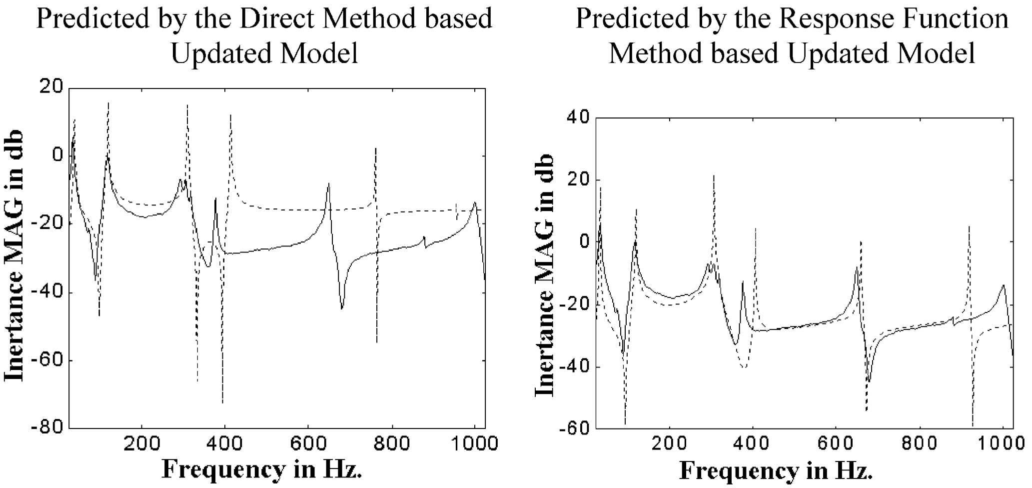

Figure 18. A comparison of the overlays of the measured modified FRF (——) and those predicted by the updated models (-----) obtained by the direct method and the response function method for the case of *beam modification*.

TABLE 8

Comparison of the natural frequency and the mode shape correlation for the beam-modified structure as predicted by the updated models based on the two methods

Mode no.	Measured frequency for the modified structure (Hz)	Predictions for the modified structure on the basis of the direct method based updated model: Frequency (Hz)	% Error	MAC-value	Predictions for the modified structure on the basis of the response function method based updated model: Frequency (Hz)	% Error	MAC-value
1	33·95	38·14	12·36	0·9823	33·66	− 0·83	0·9892
2	117·30	118·25	0·81	0·9927	120·75	2·94	0·9929
3	309·98	312·00	0·65	0·9201	307·78	− 0·70	0·8641
4	376·89	413·14	9·61	0·3661	405·23	7·51	0·7283
5	648·34	761·67	17·48	0·9575	659·35	1·69	0·9845
6	1001·21	957·34	− 4·38	0·9317	919·00	− 8·21	0·9519
7	1489·98	1126·83	− 24·37	0·8259	1474·08	− 1·06	0·9593

the simulated study discussed in section 4. On the basis of these results it thus appears that the correction of the FE model in the case of RFM based updating has been able to reduce the error in a physical sense. This is primarily on account of the flexibility, which the iterative methods offer in the selection of the updating parameters. This flexibility allows bringing an element of engineering judgement, about the possible sources of the modelling inaccuracies, for deciding as to which parameters should be corrected.

Therefore, it can be said that the modified dynamic behavior due to potential structural modifications can be predicted with reasonable accuracy on the basis of updated models. However, every updated model, which has a good correlation with the test data, may not necessarily be equally good in making predictions of the modified behavior. It appears that those updated models that have been able to minimize modelling inaccuracies in the FE model can predict more reliably.

6. CONCLUSION

The suitability of updated models for performing dynamic design has been investigated in this paper. Dynamic design can be performed successfully at the computer level if updated models can be used to predict accurately the effects of structural modifications. Towards this end, a simulated study on a beam structure is performed involving the cases of complete, incomplete and incomplete and noisy simulated experimental data. Updated models for these cases are obtained by using a direct method and an iterative method of model updating based on the FRF data. It is seen that updated models based on both the methods have been generally successful in predicting the effects of structural modifications. But the predictions made by the iterative method based updated model seem to be particularly more accurate for both the cases of mass and beam modification. The difference in the predictive capability of these updated models is traced by an analysis of the corrections that were made to the FE model at the updating stage. It is observed that in the case of iterative method the parameters that were in want of correction were updated and the corrections were made, in terms of their locations and amount, only where they were actually required, while in the case of the direct method, the changes made in the FE model matrices, to achieve a match between the test and the updated FE model modal data, are found to be not able to minimize significantly the modelling inaccuracies.

The case of an F-shape structure then is considered for carrying out the study in the presence of actual measured data. The direct method produced an updated model that exactly reproduces the identified modal data. An updated model using the iterative method of model updating is obtained by updating the parameters related to the joints present in the structure. These updated models are then used to predict the changes in the dynamic characteristics brought about by a mass and then a beam structural modification and are compared with the actual measured changes. Results of the updated models based predictions show a trend that is quite similar to that observed in the simulated study. It is found that the predictions based on the iterative method based updated model are reasonably accurate and, therefore, this updated model can be used with reasonable accuracy to perform dynamic design. The predictions on the basis of the direct method based updated model are found to be reasonably accurate in the lower portion of the updating frequency range but the predictions are in a significant error in the remaining portion of the updating frequency range. On the basis of these studies, it is concluded that the updated models that are closer to the structure physically are likely to perform better in predicting the effects of structural modification. This can be achieved if by the process of updating one is able to minimize the modelling inaccuracies present in an FE model.

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